Vertical Mergers with Input Substitution:
Double Marginalization, Foreclosure and Welfare

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Abstract

We consider differentiated duopolists that face symmetric linear demands and produce using identical or different Cobb-Douglas technologies with a monopolized input and a competitively supplied input. A merger between the input monopolist and either firm eliminates double marginalization but—unlike with fixed-proportions technologies in the same setting—can lead to foreclosure and reduce consumer welfare and total welfare. The same can occur under a CES technology with greater input substitutability than Cobb-Douglas. When firms use identical Cobb-Douglas technologies, the merged firm raises the rival's cost by more, and the welfare effects are worse, when the input it controls constitutes a low rather than high share of downstream input costs. If that share is sufficiently low then consumer welfare and total welfare decline, while rising elsewhere despite foreclosure. With different Cobb-Douglas technologies, the input monopolist may foreclose completely either firm pre-merger. A merger's welfare effects then can be non-monotonic in the monopoly input's share of a firm's costs.

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1. Introduction

Vertical mergers combine firms or assets at different stages of the supply chain, such as an upstream input supplier and a downstream output producer. Unlike horizontal mergers, the parties to a vertical merger are not competitors but instead engage in complementary economic activities. Examples of prominent vertical mergers include Comcast/NBCU (2011), AT&T/Time Warner (2018), and CVS/Aetna (2019).[1] Competition policy recognizes that vertical mergers can generate efficiencies, by alleviating transaction costs between the parties or reducing pricing distortions. In the familiar case of successive monopoly—e.g., a monopolist manufacturer selling to a monopolist distributor under linear pricing—vertical integration benefits the merging firms and consumers by expanding output (Spengler, 1950) due to elimination of double marginalization (EDM).[2] This successive monopoly scenario underlies common perceptions that vertical mergers are beneficial.

However, when one of the merging parties commands substantial market power and initially transacts with rivals of its merger partner a major competitive concern is foreclosure. The merged firm may worsen the access terms to its assets for rivals compared to the terms that rivals would enjoy absent the merger, and thereby harm competition either "upstream" (via *customer foreclosure*) or "downstream" (via *input foreclosure*).[3]

To sharpen the contrast with the beneficial scenario, consider an alternative hypothetical where firms 1 and 2 produce differentiated substitutes goods using different technologies, and only firm 2 utilizes an input sold by a monopolist supplier, firm S. A merger of firm S with firm 1 yields no EDM.[4] Instead, the merger generates two anti-competitive pricing incentives for the merged firm M. One is the familiar foreclosure or *raising rivals' costs* (RRC) *incentive*: raise the input price to firm 2 in order to induce a price rise for good 2, which increases profits from firm M's sales of good 1. Another is the *Chen incentive*: raise the price of good 1 to increase sales of good 2, hence firm M's

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[1] See, e.g., Rogerson (2014); Memorandum Opinion of Judge Richard J. Leon in *United States v. AT&T and Time Warner*, June 12, 2018, at https://ecf.dcd.uscourts.gov/cgi-bin/show_public_doc?2017cv2511-146; and Memorandum Opinion of Judge Richard J. Leon in *United States v. CVS and Aetna*, September 4, 2019, at https://ecf.dcd.uscourts.gov/cgi-bin/show_public_doc?2018cv2340-135.

[2] EDM refers to the incentive of the merged firm to reduce its output price because it obtains the input internally at cost.

[3] See, e.g., supra note 1, and section 4 of the *Vertical Merger Guidelines* issued by the U.S. Department of Justice and Federal Trade Commission, June 30, 2020 (hereafter, "VMG"). See also Zenger (2020), who discusses the interaction between foreclosure and EDM incentives, and examples from European Commission cases.

[4] The VMG call such a merger "diagonal" as it combines "firms or assets at different stages of competing supply chains" (VMG, p.1; see also Example 7).

profitable input sales to firm 2, a gain that was ignored by the independent firm 1.[5] Although the merging firms operate at different stages of the economic value chain, in this scenario the merger is more accurately characterized as purely horizontal as judged by the incentives it creates. RRC is akin to forcing firm 2 to join in downstream collusion. The Chen incentive is akin to firm 1 acquiring a partial ownership interest in its rival, because firm 1 now shares in firm 2's gross profits via input sales to firm 2 by firm 1's merger partner.[6] Both incentives push downstream prices higher.

Vertical mergers that draw antitrust scrutiny typically lie between the above polar cases: the merging supplier initially sells both to its downstream merger partner and to the partner's rival(s). Thus, a 'vertical' merger in oligopoly is best characterized as both vertical and horizontal, exhibiting all three incentives: RRC, EDM and Chen.[7] These three incentives, which can interact in complex ways, jointly determine the post-merger equilibrium values of the relevant variables. This paper addresses the welfare effects of such vertical mergers when downstream firms can substitute (imperfectly) away from the input available only from the merging supplier.

We consider an input monopolist (firm S) that may supply to two downstream producers (firms 1 and 2) of differentiated substitute goods. In our main model both producers use constant-returns Cobb-Douglas technologies with two inputs: the monopoly input X and a competitively supplied input whose price is given. The parameter α_i is the elasticity of firm *i*'s output with respect to input X. Firm S merges with firm 1. The model nests the two polar cases when input X is needed (i) only by the merger partner ($\alpha_1 > 0, \alpha_2 = 0$) or (ii) only by the rival ($\alpha_1 = 0, \alpha_2 > 0$), with the other firm utilizing only the competitive input. It also includes the more interesting mixed cases ($\alpha_1 > 0$ and $\alpha_1 > 0$). For tractability we consider symmetric linear demands for the differentiated goods.

Pre-merger, the monopoly input is sold at linear prices (implying double marginalization) under public offers. These assumptions simplify the analysis and let us focus on the role of input

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[5] To our knowledge, this incentive was first identified in Chen (2001). It is not mentioned in the VMG but was noted in the draft issued for public comment, January 2020, p.7: "[I]f the merged firm raises its price in the downstream market, downstream rivals may increase their sales, which could increase their demand for inputs from the merged firm's upstream business."

[6] Moresi and Salop (2020) use the analogy to partial ownership interests for the purpose of quantifying the increase in "effective concentration" from vertical mergers that raise input foreclosure concerns.

[7] We view the EDM incentive as referring to the merged firm setting its output price so as to internalize the effect on its profit from inputs used by its downstream unit (which firm 1 ignored pre-merger), while the Chen incentive internalizes the profit from inputs sold to the rival. The EDM and Chen incentives push the merged firm's output price in opposite directions.

substitution.[8] We solve the model numerically in Wolfram Mathematica using a uniform grid for (α_1, α_2) with a step size of 0.005. The code is available upon request.

With linear demands for the differentiated products, a merger in our setting would increase consumer welfare and total welfare if inputs were not substitutable but instead were used in fixed proportions with output (see Lu et al., 2007 and Section 3.1 below). Fixed-proportions production is a limiting case in our model ($\alpha_1 = \alpha_2 = 1$). One might conjecture that a merger will increase welfare also when input substitution is possible, as this could only reduce the merged firm's *ability* to foreclose rivals. However, we refute this conjecture: we show that a merger can reduce (consumer and total) welfare when firms produce with identical or different Cobb-Douglas technologies. As a robustness check, we also consider a common CES technology with greater input substitutability than Cobb-Douglas and where each input individually is not essential. The merger can reduce welfare also in that setting. Intuitively, with input substitution the rival is a less attractive input customer than under fixed-proportions, which magnifies the merged firm's *incentive* to divert output from the rival to its own good by raising the input price.

When firms use the same Cobb-Douglas technology, the welfare effects of a merger are worse if the input controlled by the merged firm is used in low rather than high intensity (accounts for a low share of downstream input costs). There are two main forces. First, when the usage intensity is lower the cost distortion eliminated for the merged firm is smaller, hence its incentive to cut its output price due to EDM is weaker. Second, the RRC incentive is stronger. The merged firm diverts more output sales from the rival to itself—via a sufficiently larger increase in the input price—because it loses a smaller input margin (per unit of the rival's output) than when the input is used with high intensity.[9]

When firms use different Cobb-Douglas technologies, the input monopolist may foreclose completely either firm pre-merger. The welfare effects of a merger then can be non-monotonic. For example, when the rival's usage intensity is low, the merger reduces welfare if the merger partner's usage intensity lies in an intermediate range but raises welfare elsewhere.

The remainder of the paper is organized as follows. Section 2 presents the model and some comparative statics for the above polar cases, varying only α_1 or only α_2. Section 3 analyzes the

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[8] The VMG (2020) predominantly discuss linear pricing. For an analysis that addresses nonlinear pricing, secret contracts, and upstream strategic competition, see Rey and Vergé (2019). Our framework is similar to Inderst and Valletti (2011), but they consider a different technology and focus on different issues.

[9] With fixed proportions (Leontief technologies), the input monopolist earns the same margin per unit of output regardless of the input proportions used by a firm.

case where the merging supplier's input is used by both firms with Cobb-Douglas technologies. Section 4 considers a common CES technology, and Section 5 concludes.

2. A Model where 'Vertical' Mergers Can Be Vertical, Horizontal, or Both

2.1 The Setting

Firms 1 and 2 sell differentiated substitute goods to consumers. Firm i produces its good using two inputs (X and Y) and a Cobb-Douglas technology with constant returns to scale:

$$Q_i = \beta_i X_i^{\alpha_i} Y_i^{1-\alpha_i} \tag{1}$$

where Q_i is the quantity of good i, X_i and Y_i are the input quantities, $\beta_i > 0$ is a scale factor, and $0 \leq \alpha_i \leq 1$ is the elasticity of output with respect to input X.

Input Y is available competitively at a price equal to 1, while input X is produced by a monopolist supplier (firm S) with marginal cost also equal to 1.[10] Input X is sold to firm i at price W_i. Cost minimization with technology (1) implies that firm i's spending on input X accounts for a fraction α_i of firm i's total spending on both inputs. In other words, α_i is the *cost share* of the monopoly input at firm i.

We assume $\beta_i = \alpha_i^{-\alpha_i}(1-\alpha_i)^{-(1-\alpha_i)}$, so that firm i's marginal cost of production is $c_i = W_i^{\alpha_i}$. After firm S merges with firm 1, the merged firm sources input X internally at a cost equal to 1, purchases input Y at a price equal to 1, and hence $c_1 = 1$ post-merger. Otherwise, firm S sets prices above marginal cost ($W_i > 1$) and thus $c_i = W_i^{\alpha_i} > 1$ if $\alpha_i > 0$. Firm i's conditional factor demand for input X is:

$$X_i = \alpha_i (1/W_i)^{1-\alpha_i} Q_i \tag{2}$$

Demand functions for the two differentiated goods are assumed to be symmetric and linear:

$$Q_i = 2 - 2P_i + P_j \tag{3}$$

where P_i is the price of good i (and $j \neq i$). Thus, the two goods are imperfect substitutes with a diversion ratio of ½. We further assume that demands are derived from a linear-quadratic utility function of a representative consumer, whose utility gives consumer welfare (CW). Total welfare

[10] If input X was available competitively (as is input Y), then the model would be symmetric with respect to both the demand for the two goods and the supply of the two inputs. It follows that input X will be more expensive than input Y (since it is supplied by a monopolist) and that the marginal cost of a downstream firm is higher if it uses input X more intensively (ceteris paribus).

(TW) is the sum of CW and the profits of firms S, 1 and 2.

The pre-merger game has several stages. First, firm S sets input prices W_1 and W_2. Second, firms 1 and 2 observe both input prices and simultaneously set output prices P_1 and P_2 to consumers. Third, firms 1 and 2 receive the order quantities Q_1 and Q_2 from consumers, purchase the required (cost-minimizing) quantities of inputs X and Y, produce, and deliver the goods. Each firm sets its price(s) to maximize its own profit: $\pi_i = (P_i - W_i^{\alpha_i})Q_i$ for firm i (i=1,2), where Q_i is given in (3), and $\pi_S = \sum_{i=1,2}(W_i - 1)X_i$ for firm S, where X_i and Q_i are given by (2) and (3).

The post-merger game is similar. The merged firm (firm M) sets the input price W_2 to firm 2. Then, firms M and 2 set output prices P_1 and P_2 to consumers, receive order quantities Q_1 and Q_2, obtain the required inputs, produce, and deliver the goods. Firm M maximizes its integrated profit: $\pi_M = (P_1 - 1)Q_1 + (W_2 - 1)X_2$.[11] As before, firm 2 maximizes $\pi_2 = (P_2 - W_2^{\alpha_2})Q_2$.

The above setting nests the two polar cases and the mixed case from the Introduction. We summarize here some results for the polar cases which help provide intuition for the mixed case.

2.2 Merging supplier's input is used only by merger partner

Let firms 1 and 2 have the Cobb-Douglas technology in (1), with $\alpha_1 > 0$ and $\alpha_2 = 0$. Here, $c_2 = 1$ since firm 2 uses only the competitive input. Figure 1 shows how the pre-merger equilibrium input price to firm 1 (W_1) and firm 1's marginal cost of good 1 (c_1) vary as functions of the parameter α_1. As α_1 falls, firm 1's demand for input X becomes less elastic: When α_1 is lower (input X accounts for a lower share of total input costs), the pass-through from W_1 to firm 1's marginal cost $c_1 = W_1^{\alpha_1}$ is lower, so a given increase in W_1 has a smaller effect on firm 1's price P_1 and output Q_1, dampening the decrease in demand for X. In response, firm S raises W_1 as α_1 falls, but c_1 still decreases. It follows that the pre-merger price P_1 decreases as α_1 falls. Firm 2's price P_2 also decreases (because with linear demands prices are strategic complements), though more slowly.

Post-merger, good 1's marginal cost is $c_1 = 1$ for any α_1, since the marginal cost of the merged firm's internally sourced input is equal to the price of the competitively supplied input, 1. The reduction in c_1 due to the merger therefore is smaller at lower α_1, i.e., when the merging supplier's input comprises a smaller share of the merger partner's cost. Because the merged firm in this

[11] This view of the behavior of an integrated firm as a centralized unit is standard in the literature. See, e.g., Chen (2001) and Arya et al. (2008). Moresi and Schwartz (2017) consider that case and an alternative scenario (Delegation) where the downstream unit acts autonomously to maximize its own profits, in which case W_1 is no longer an irrelevant internal transfer price.

scenario does not supply input X to firm 2, it treats $c_1 = 1$ as its relevant marginal cost for good 1.[12] It sets P_1 accordingly, so P_1 is independent of α_1 post-merger. Thus, the *reduction* in P_1 due to elimination of double marginalization also is smaller at lower α_1: *The EDM effect is weaker when α_1 is smaller*.

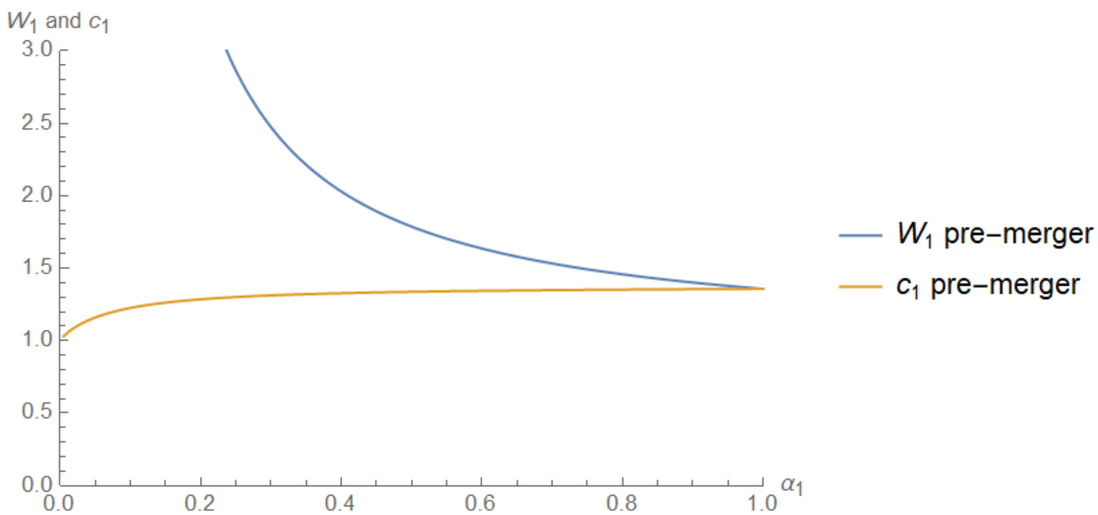

Figure 1: Pre-merger W_1 and c_1 as functions of α_1

The welfare effects in this case are straightforward. Due to EDM, the merger always (for all α_1) benefits the merging firms, consumers and total welfare, while harming the rival. The changes are potentially large. Consumer welfare and the merging firms' profits each can rise by over 40%, total welfare by over 15%, while the rival's profit can fall by over 20%.

2.3 Merging supplier's input is used only by merger partner's rival

Suppose $\alpha_1 = 0$ and $\alpha_2 > 0$. In this scenario there is no EDM since firm 1 does not use the input controlled by its merger partner, firm S. While the merger ostensibly is still 'vertical' because firm *S* and firm 1 operate at different stages of the supply chain, the incentives generated are purely horizontal in nature and the merger will raises all downstream prices.

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[12] If the merged firm also supplies input X to the rival (Section 3), its marginal cost of supplying good 1 to consumers includes the resource cost $c_1 = 1$ and an opportunity cost—the input profits lost per additional unit sold of good 1 due to diversion of sales from good 2. Internalization of this opportunity cost when setting P_1 is the "Chen incentive."

Pre-merger, firm S earns profit only from input sales to firm 2. After firms *S* and 1 merge, the merged firm M has two potential sources of profit: input sales to firm 2 and sales of its own good 1. These profit sources are substitutes for firm M because goods 1 and 2 are substitutes. Therefore, relative to the individual incentives of firm S and firm 1 pre-merger, firm M has two anti-competitive pricing incentives noted earlier: RRC and Chen.

We denote an increase in W_2 as RRC or "foreclosure" and find two sub-cases: *partial foreclosure*, where firm 2 remains active but pays a higher input price; and *complete foreclosure*, where firm 2 is driven completely out of the market.[13] For α_2 above a threshold ($\alpha_2 > 0.145$), firm M continues to supply its input X to firm 2, though at a higher W_2 than pre-merger. As α_2 decreases from 1 in this range, firm 2 becomes a less attractive input customer and firm M raises W_2 by a greater percent compared to the pre-merger level. For α_2 below the threshold ($\alpha_2 < 0.145$), firm M sets W_2 prohibitively high and firm 2 is foreclosed completely. In this range, input *X* would earn such a low share of firm 2's total input spending that firm M prefers to forgo all input sales and earn profits solely from selling increased quantities of its own good 1. Thus, foreclosure becomes more attractive to the merged firm when the customer/rival is a less important input customer.[14]

Figure 2 shows the percent increase in W_2 post-merger as a function of α_2 in a range where firm 2 remains active (partial foreclosure). Figure 3 contrasts the resulting behavior of firm 2's marginal cost c_2 pre-merger versus post-merger as functions of α_2.[15] As α_2 decreases, firm S raises the input price W_2 but c_2 still decreases because firm 2 uses fewer units of the higher-priced input X. Post-merger, firm M raises W_2 more rapidly as α_2 decreases (Figure 2), and by enough that c_2 *increases* as α_2 decreases (Figure 3).[16] Perhaps surprisingly, therefore, the *RRC effect is stronger when α_2 is*

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[13] The VMG (2020, Section 4.a) use "foreclosure" to denote refusal to supply the input, even if the rival remains active, and "raising rivals' costs" to denote an increase in the input price (presumably to a non-prohibitive level, since a prohibitive price increase would amount to refusal to supply).

[14] This finding is reminiscent of O'Brien and Salop's (2000) analysis of the competitive effects of partial ownership by competing firms. The greatest harm arises when the partial owner has voting control over the other firm's pricing but holds only a small ownership share, hence does not lose much by handicapping that firm. Here, the merged firm controls firm 2's pricing through the input price, and its loss from handicapping firm 2 is lower when firm 2's spends a smaller share of its input costs on the merged firm's input.

[15] Pre-merger, the behavior of c_2 as a function of α_2 is identical to c_1 as a function of α_1 from Figure 1. In both scenarios, firm S initially supplies to a single firm that produces with the technology given in (1) and competes against a rival that uses only the competitive input.

[16] Firm S being an unconstrained monopolist is important here. If the price of input X were capped, say because X is available from a higher-cost source at a fixed price not much above firm S's marginal cost, then firm S would be unable to raise W_2 enough to outweigh the direct effect on c_2 of a decrease in α_2.

smaller — the increase in the rival's cost due to the merger is larger when the monopoly input controlled by the merged firm constitutes a *smaller* share of the rival's cost.[17]

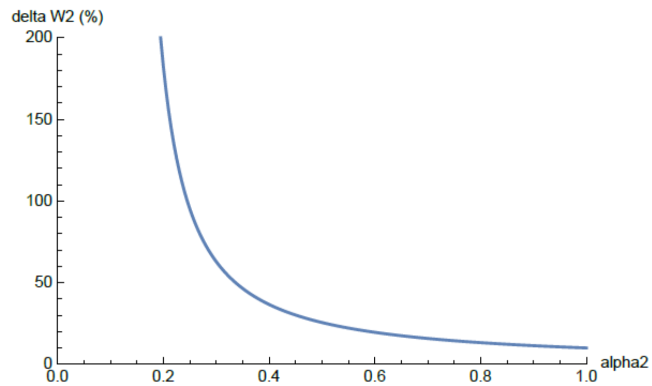

Figure 2: $\Delta W_2(\%)$ as function of α_2

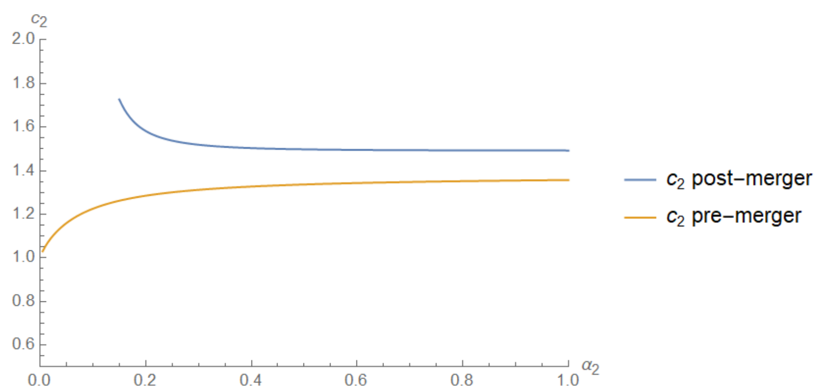

Figure 3: Firm 2's cost c_2 pre-merger and post-merger as functions of α_2

Further intuition for why RRC is stronger when α_2 is smaller can be gleaned by considering how the pre-merger equilibrium changes as α_2 falls below 1.[18] Table 1 shows the equilibrium values of relevant variables pre-merger and post-merger. Firm S's margin per unit of firm 2's output, $(W_2 - 1)(X_2/Q_2)$, declines as α_2 falls, due to the fall in X_2/Q_2. Therefore, starting at the pre-merger equilibrium values, the loss of input profits to firm M if firm 2's output is reduced by one

[17] We will find the same pattern in Section 3 where both downstream firms use the monopoly input and in equal intensity ($\alpha_1 = \alpha_2 \in (0, 1]$).

[18] In the spirit of UPP analysis, we start at the pre-merger equilibrium outcome and consider the "first round" pricing incentives following a merger.

unit decreases as α_2 falls. Firm M's gain from selling an additional unit of good 1, $(P_1 - 1)$, also declines (because pre-merger, when α_2 falls c_2 decreases, which reduces P_2 hence also P_1), but more slowly than the loss. Since the diversion ratio from good 2 to good 1 is constant with linear demands, the merged firm finds it increasingly attractive to divert output from the rival's good to its own as α_2 falls. It implements such greater diversion by inducing a larger increase in c_2 when α_2 is smaller through successively larger increases in W_2.

	PRE-MERGER					POST-MERGER		CHANGE	
α_2	W_2	c_2	$\frac{X_2}{Q_2}$	$(W_2 - 1)\frac{X_2}{Q_2}$	P_1	W_2	c_2	ΔW_2	Δc_2
1	1.357	1.357	1.00	0.357	1.381	1.492	1.492	0.135	0.135
0.8	1.458	1.352	0.74	0.339	1.380	1.651	1.493	0.193	0.141
0.6	1.636	1.343	0.49	0.312	1.379	1.955	1.495	0.319	0.152

Table 1: Understanding RRC when only the rival uses the monopoly input

Prices of both goods rise post-merger, as shown in Figure 4.[19] The percentage increase in the rival's price P_2 diminishes as α_2 rises above 0.145, tracking the RRC pattern from Figure 3. The increase in P_1 also diminishes initially as α_2 rises above 0.145, but eventually becomes larger—despite the continued decline in ΔP_2(%). This illustrates the *Chen incentive*: as firm 2 uses the merging supplier's input more intensively (α_2 increases), the merged firm stimulates sales of the rival's good not only by reducing the increase in the input price W_2 but also by magnifying the increase in the price of its own good P_1.[20]

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[19] In Figure 4, there is a kink at α_2 = 0.145, where total foreclosure turns into partial foreclosure. For $\alpha_2 \leq$ 0.145, we set P_2 equal to the 'choke price' of firm 2's residual demand conditional on P_1.

[20] The Chen and RRC incentives interact and jointly determine the behavior of P_1. For $\alpha_2 \geq 0.15$, firm M increases P_1 for two reasons: diversion of output sales to the rival increases input sales to the rival, and partial foreclosure makes good 1's demand less elastic. For $\alpha_2 \leq 0.145$, the first reason vanishes due to complete foreclosure, but the second reason becomes stronger.

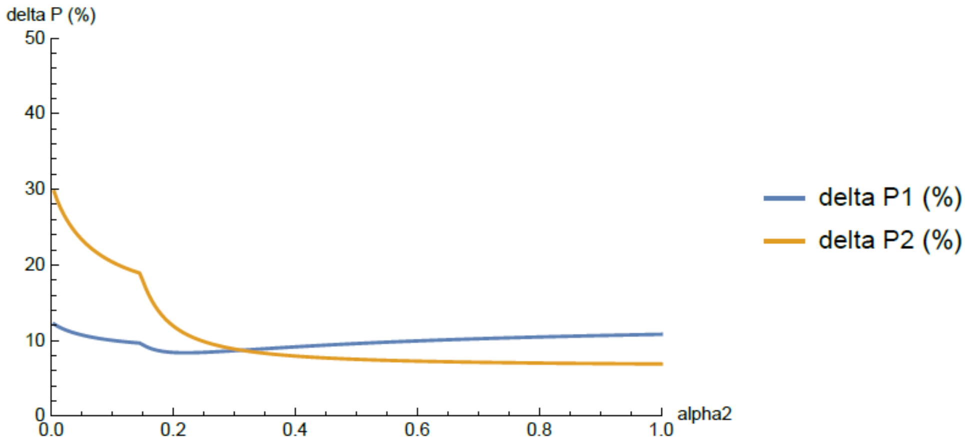

Figure 4: ΔP_1 (%) and ΔP_2 (%) as functions of α_2

The merger reduces the rival's profit by at least 35% under partial foreclosure and by 100% with complete foreclosure, while joint profits of the merging firms rise between 10% and 55%. Total welfare declines between (approximately) 15% and 35%, while consumer welfare declines between 35% and 55%.

3 Both Producers Use the Merging Supplier's Input

3.1 Same Cobb-Douglas Technology

Suppose firms 1 and 2 have the same technology: $\alpha_1 = \alpha_2 = \alpha \in (0,1]$. Variations in the common α do not affect the marginal cost of good 1 post-merger ($c_1 = 1$), but will affect firm 2's cost as well as the pre-merger equilibrium. When $\alpha = 1$, both firms use only the input supplied by firm S, which then is used in *fixed proportions* with output. In that case, and for a broad class of linear demands downstream, Lu et al. (2007) showed that a vertical merger of an input monopolist and a downstream duopolist raises consumer welfare (CW) and total welfare (TW): it lowers the prices of both goods, and even lowers the input price to the rival.[21] The last column in Table 2 shows the merger effects in our model when $\alpha = 1$.[22] We will discuss the intuition for all the patterns in Table 2 shortly.

[21] That result does not extend to logit demand or upstream bargaining. See Das Varma and De Stefano (2020).

[22] A merger raises welfare also in a related setting with fixed proportions analyzed by Agkün et al. (2020). They consider a monopolist seller of one component that is used with a second component to form a system assembled by consumers. Differentiated duopolists facing linear demands sell the second component directly

Input X's cost share (α)	0.05	0.1	0.25	0.5	1
Input price to firm 2 (W_2)	-----	-----	42.8%	4.46%	-0.51%
Marginal cost of good 2 (c_2)	-----	-----	9.31%	2.21%	-0.51%
Price of good 2 (P_2)	-----	-----	2.28%	-1.04%	-2.27%
Quantity of good 2 (Q_2)	-100%	-100%	-47.7%	-27.8%	-18.2%
Marginal cost of good 1 (c_1)	-16.9%	-22.7%	-28.9%	-31.8%	-33.3%
Price of good 1 (P_1)	2.1%	-1.88%	-7.19%	-8.11%	-8.18%
Quantity of good 1 (Q_1)	41.1%	59.1%	67.6%	70.0%	70.5%
Consumer welfare (CW)	-33.6%	-15.6%	31.9%	54.7%	65.7%
Total welfare (TW)	-10.3%	1.02%	18.7%	21.1%	18.2%

Table 2: Merger effects when firms 1 and 2 use the same Cobb-Douglas technology

Input substitution ($0 < \alpha < 1$), whereby inputs are used in *variable proportions* with output, introduces rich possibilities not captured by the fixed-proportions case. In Table 2, consider the merger effects as α declines from 1 to 0.05. When $\alpha = 0.5$, firm M raises the input price W_2. The rival's price P_2 still falls, however, due to the expected fall in P_1, triggered by EDM. When $\alpha = 0.25$, W_2 increases by much more, causing P_2 to rise. In both cases ($\alpha = 0.5$ or 0.25), CW and TW increase despite partial foreclosure of firm 2 (rise in W_2), due to the reduction in P_1 and expansion in Q_1.[23] For α below about 0.15, firm M chooses complete foreclosure of firm 2. When $\alpha = 0.1$, firm M still reduces P_1, but now CW falls (due to the loss of good 2), though TW still rises (due to elimination of cost distortion in production of good 1). Complete foreclosure occurs also when $\alpha = 0.0.5$, and this time firm M raises P_1, leading to a reduction in both CW and TW.

Consumer welfare and total welfare each can rise or fall substantially. For CW, the range is from about – 55% at α near 0 to over 60% at $\alpha = 1$, while for TW the range is from –35% at α near 0 to 20% at α around 0.3. As expected, joint profits of the merging firms always rise, with the gain

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to consumers, and pre-merger all three prices are set simultaneously (whereas in our setting the monopoly price is set first). A merger between the monopolist and either duopolist leads to an *increase* in the monopoly component's price (unlike our decrease in W_2), but still lowers both systems' prices.

[23] With partial foreclosure, the increase in c_2 is smaller than the increase in W_2 because of input substitution. For all α between 0 and 1, c_1 decreases post-merger. The decrease is smaller at lower α because post-merger c_1 = 1 regardless of α, whereas pre-merger c_1 is lower when α is lower (recall discussion of Figure 1, which applied when only firm 1 used input X, but the same logic holds when firms have the same α).

ranging from about 10% to almost 90%. The rival's profit falls by at least 30% and up to 100% with complete foreclosure.[24] Both magnitudes, merging firms' gain and the rival's loss, are larger at low α than at high α, corresponding to the finding that the extent of foreclosure is greater at low α.

The patterns in Table 2 can be further understood as follows. First, consider the merged firm's incentives regarding P_1, starting at the pre-merger equilibrium values of P_1, P_2, W_1 and W_2. Express firm M's profit as $\pi_M = \pi_1 + (W_1 - c)X_1 + (W_2 - c)X_2$, where π_1 is the profit of former firm 1, c is the marginal cost of input X, and $\partial\pi_1/\partial P_1 = 0$ at the pre-merger equilibrium. Thus,

$$\frac{\partial \pi_M}{\partial P_1} = \underbrace{(W_1 - c)\left(\frac{\partial X_1}{\partial Q_1}\frac{\partial Q_1}{\partial P_1}\right)}_{EDM\ incentive} + \underbrace{(W_2 - c)\left(\frac{\partial X_2}{\partial Q_2}\frac{\partial Q_2}{\partial P_1}\right)}_{Chen\ incentive}.$$

Pre-merger, $W_1 = W_2 = W^* > c$, and $\partial X_1/\partial Q_1 = \partial X_2/\partial Q_2$ since the firms have identical technologies and we are starting from a symmetric equilibrium. Thus,

$$\frac{\partial \pi_M}{\partial P_1} = (W^* - c)\frac{\partial X_i}{\partial Q_i}\left[\frac{\partial Q_1}{\partial P_1} + \frac{\partial Q_2}{\partial P_1}\right] < 0.$$

The inequality holds because with linear demands the term in square brackets is constant and negative (–1 in our demand system (3))—own-price effect outweighs cross-price effect—and the other two terms are positive. Thus, the EDM incentive dominates the Chen incentive, so firm M wants to reduce P_1 for all $\alpha \in (0,1]$ (holding all other prices constant).

The net EDM incentive (EDM net of Chen) measured by $\partial\pi_M/\partial P_1$ is strongest when α =1. As α falls, there are two opposing effects on this net incentive: (i) W^* rises pre-merger, hence (W^* – c) rises; but (ii) from (2), $\partial X_i/\partial Q_i = \alpha(1/W^*)^{1-\alpha}$ falls, since $W^* > 1$. That is, when α is lower, an increase in output leads to a smaller increase in demand for input X. Effect (ii) dominates, so the merged firm's incentive to reduce P_1 (evaluated at the pre-merger equilibrium) diminishes when its input is used less intensively.

Now consider the interaction between the RRC and net EDM incentives. For α near 1, the net EDM incentive is strong, leading to a large decrease in P_1, which reduces firm 2's output and demand for input X, making the latter more elastic. This effect is strong enough that firm M prefers to reduce the input price W_2, despite the RRC incentive. As α decreases, starting at the pre-merger

[24] The rival's large loss is relevant for policy if the rival has significant fixed costs and one is concerned about adverse long-term effects on market structure. However, claims of harm to rivals generally should be treated with healthy skepticism, to avoid condemning mergers that harm rivals but benefit consumers.

equilibrium values, the net EDM incentive weakens whereas the RRC incentive strengthens (as discussed shortly), explaining the patterns in Table 2.

In particular, as α falls, W_2 eventually increases above its pre-merger level and the percentage increase becomes larger, ultimately leading to complete foreclosure. The pattern is similar to that in Figure 2 (where only the rival used input X), except that for high values of the common α ($0.84 \leq \alpha \leq 1$) firm M now reduces W_2 below the pre-merger level due to the EDM incentive. Driven by the change in W_2, post-merger c_2 increases as α falls, whereas pre-merger c_2 decreases as α falls.[25]

Thus, the merger-induced Δc_2 is larger when α is smaller.[26] The logic is the same as in Section 2.3, where only the rival used input X, and Table 3 is the counterpart to Table 1. As α falls, firm M's loss of input profits if firm 2's output is reduced by one unit, $(W_2 - 1)(X_2/Q_2)$, declines faster than M's gain from selling another unit of good 1, $(P_1 - 1)$. Therefore, firm M diverts more output from the rival's good to its own as α fall, by increasingly raising c_2 through larger increases in W_2.

	PRE-MERGER					POST-MERGER		CHANGE	
α	W_2	c_2	$\frac{X_2}{Q_2}$	$(W_2 - 1)\frac{X_2}{Q_2}$	P_1	W_2	c_2	ΔW_2	Δc_2
1	1.500	1.500	1.00	0.500	1.667	1.492	1.492	–0.008	–0.008
0.8	1.648	1.491	0.72	0.467	1.661	1.651	1.493	0.003	0.002
0.6	1.915	1.477	0.46	0.421	1.651	1.955	1.495	0.040	0.018

Table 3: Understanding RRC when both firms have the same Cobb-Douglas technology

3.2 Different Cobb-Douglas Technologies

New possibilities emerge when firms 1 and 2 have different technologies ($\alpha_1 \neq \alpha_2$) and both use input X. The pre-merger equilibrium will be asymmetric. And unlike in the polar cases, firm S pre-merger is able to foreclose completely either firm by refusing to supply its input. If α_1 and α_2 are strictly positive but different enough, firm S pre-merger will foreclose the low user of its input. The merger effects thus can be quite complex.

[25] The patterns are the same as in Figure 3, where only α_2 varied, except that c_2 there always rose post-merger, whereas now c_2 falls post-merger at high α due to the decrease in W_2.

[26] Moresi and Schwartz (2020) cite this finding for identical Cobb-Douglas technologies (common α) to caution against a safe-harbor for vertical mergers based on the merging supplier's input accounting for a low share of rivals' input costs. (The same pattern was found, and its logic was explained, in Section 2.3 when only the merger partner's rival uses firm S's input.)

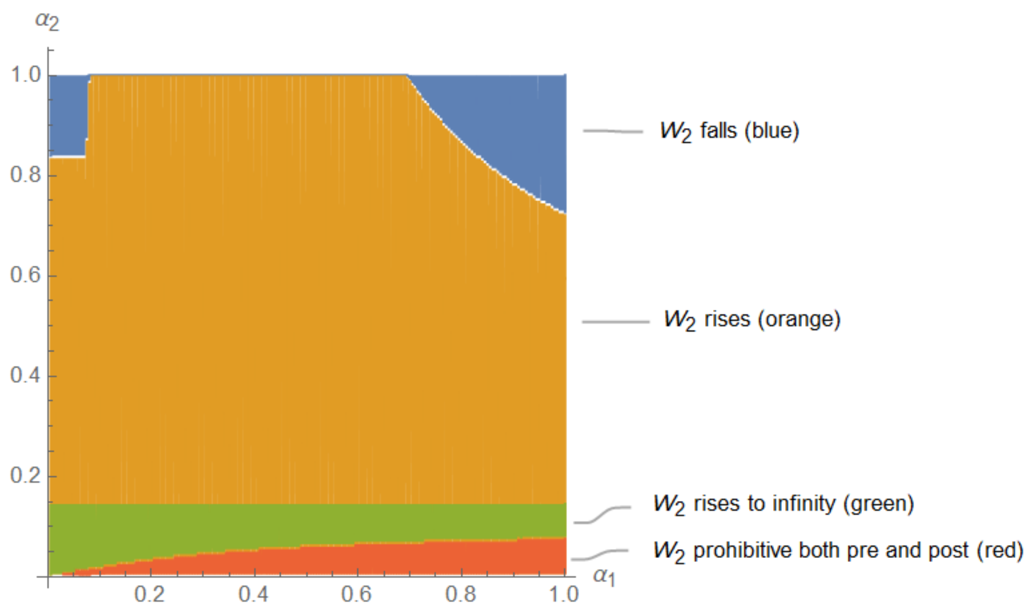

Figure 5: Regions where input price W_2 falls or rises

Refer to Figure 5. First, there is a discontinuity at $\alpha_2 = 0$ since firm M can foreclose completely firm 2 when $\alpha_2 > 0$ by not supplying its input or charging a prohibitive price for it, but cannot when $\alpha_2 = 0$.

Second, when $\alpha_1 > \alpha_2 > 0$ and α_2 is sufficiently small, firm S elects to "marginalize" firm 2 *pre-merger*—charge an input price that is just high enough to drive firm 2's output down to zero. Intuitively, when firm S obtains much greater input sales per unit output of firm 1 than of firm 2, it prefers to divert maximal output to firm 1 by raising W_2 while maintaining potential competition from firm 2 in order to cap firm 1's price (raising W_2 any higher, hence firm 2's cost, would allow firm 1 to raise P_1 thereby reducing input sales to firm 1). That occurs in the red region in Figure 5. (There is a symmetric region, not shown, where firm S "marginalizes" firm 1 pre-merger.)

Third, post-merger $c_1 = 1$ for any α_1, so the post-merger equilibrium depends only on α_2 and complete foreclosure occurs for $0 < \alpha_2 \leq 0.145$. The merger thus induces *additional* complete foreclosure in the green region in Figure 5. For $\alpha_2 \geq 0.15$, complete foreclosure never occurs, but partial foreclosure can occur as shown by the orange region in Figure 5.

For high α_2, the change in W_2 is non-monotonic in α_1 because the EDM effect is non-monotonic. Consider for example $\alpha_2 = 0.9$ in Figure 5. When $\alpha_1 = 0$, we are in the polar case where the monopolized input is used only by firm 2: post-merger there is no EDM and there is partial foreclosure as firm M raises W_2. Figure 5 does not show that because of a discontinuity at $\alpha_1 = 0$: for $0 < \alpha_1 < 0.075$, firm S "marginalizes" firm 1 pre-merger (i.e., charges firm 1 an input price W_1

that is sufficiently high to drive firm 1's output down to exactly zero) and charges a relatively high W_2, while post-merger firm M *enters* the output market with good 1—which is effectively a very large EDM effect. The entry of good 1 reduces firm 2's input demand and makes it sufficiently more elastic that firm M chooses to reduce W_2 despite the RRC incentive. A post-merger reduction in W_2 also occurs for $\alpha_1 > 0.77$, because, again, there is a strong EDM effect that outweighs RRC, although here the strong EDM effect is due to good 1 using input X with high intensity (not to good 1 entering the market). For intermediate α_1, the RRC incentive dominates and W_2 rises.

As expected, the merger always increases the joint profits of the merging firms and harms the rival, except when the rival was "marginalized" pre-merger. Figure 6 shows the regions where CW and TW rise or fall. The merger lowers CW only for relatively low values of α_1 and α_2 (orange region) and lowers TW in a smaller (green) region. Both regions are small subsets of the region where the merger raises W_2 (compare with Figure 5)—providing an important reminder that harm to competitors does not imply harm to competition.

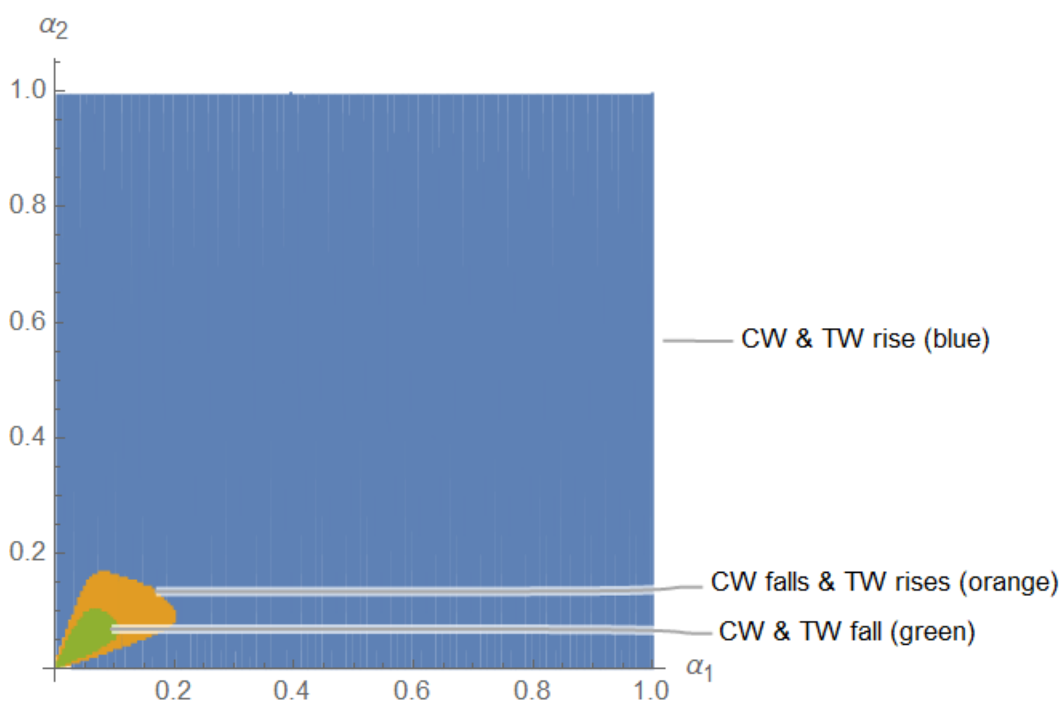

Figure 6: Regions where consumer and total welfare rise or fall

Interestingly, for low α_1 and α_2, the welfare effects are non-monotonic as one of these parameters varies holding the other fixed. For example, consider $\alpha_2 = 0.145$, so firm 2 is foreclosed completely only post-merger. If $0.05 \leq \alpha_1 \leq 0.16$, CW falls post-merger: the EDM effect does not reduce P_1 enough to offset consumers' harm from losing good 2.[27] For $\alpha_1 \geq 0.165$, the EDM effect is stronger and CW rises. For $0 < \alpha_1 \leq 0.045$, the EDM effect is stronger as well, since firm S drove firm 1's output to zero whereas the merged firm discretely increases that output, and CW rises. Intuitively, consumers purchase only one good both pre-merger and post-merger, but post-merger the price is lower due to EDM.

4. CES technology

With Cobb-Douglas technology, each input is essential. To test the role of this assumption, we consider again the symmetric case with $\alpha_1 = \alpha_2 = \alpha \in (0,1]$, and replace Cobb-Douglas by a CES technology with greater elasticity of substitution, 2 instead of 1 in Cobb-Douglas:

$$Q_i = \beta(\alpha X_i^{1/2} + (1-\alpha)Y_i^{1/2})^2 \qquad (4)$$

where $0 < \alpha \leq 1$ is a share parameter and $\beta = 1/(\alpha^2 + (1-\alpha)^2)$ is a scale factor. It follows that the cost share of input X at firm i (denoted s_i) and firm i's marginal cost are given by:[28]

$$s_i = \frac{1}{1 + W_i/\phi} \qquad \text{and} \qquad c_i = \frac{1+\phi}{1+\phi/W_i} \qquad (5)$$

where $\phi = (\alpha/(1-\alpha))^2$. As $W_i \to \infty$, $c_i \to 1 + \phi$, which is *finite* if $\alpha < 1$ because input X is not essential if $\alpha < 1$. Thus, unlike for Cobb-Douglas, firm M can no longer raise c_2 indefinitely by raising W_2 post-merger. As before, demand is given by (3).

The merger always increases joint profits of the merging firms and reduces the rival's profit (to zero when there is complete foreclosure). Table 4 illustrates the other merger effects. The last column in Table 4 corresponds to the case where both firms use only the input supplied by firm S, and thus is identical to the last column in Table 2. The other columns show several interesting differences from the symmetric Cobb-Douglas case in Table 2, as well as commonalities.

[27] In the symmetric case ($\alpha_1 = \alpha_2 = \alpha$), CW falls only if firm 2 is completely foreclosed ($\alpha \leq 0.145$) and the same is true with asymmetry if $\alpha_1 > \alpha_2$. However, if $\alpha_1 < \alpha_2$, there is a (small) region where CW falls even though firm 2 remains active post-merger (e.g. (α_1, α_2)=(0.12, 0.16)).

[28] For Cobb-Douglas, we have $s_i = \alpha$. For the CES technology in (4), s_i is decreasing in W_i and increasing in α.

Firm S's cost share (s)	0.1	0.25	0.3	0.5	1
Input price to firm 2	46.2%	-----	-----	10.3%	-0.51%
Marginal cost of good 2	3.27%	33.7%	43.7%	4.91%	-0.51%
Price of good 2	0.60%	12.7%	-----	0.35%	-2.27%
Quantity of good 2	-9.33%	-96.7%	-100%	-32.8%	-18.2%
Marginal cost of good 1	-9.78%	-21.2%	-23.8%	-29.3%	-33.3%
Price of good 1	-2.74%	-5.74%	-2.71%	-7.26%	-8.18%
Quantity of good 1	14.4%	75.1%	63.7%	61.2%	70.5%
Consumer welfare	5.58%	4.15%	-10.7%	37.9%	65.7%
Total welfare	2.83%	-3.89%	-7.07%	16.8%	18.2%

Table 4: Merger effects when firms 1 and 2 use the same CES technology

First, unlike with Cobb-Douglas, refusal to deal does not imply complete foreclosure. For example, when firm S's cost share is $s = 0.25$, the merged firm refuses to sell its input to firm 2, and firm 2's output falls substantially (by 96.7%) but not to zero because firm 2's marginal cost remains non-prohibitive. Complete foreclosure can still occur, however, for example when $s = 0.3$: firm 2's output is driven to zero by the joint impact of a higher input price and a lower downstream price for firm M due to EDM.

Second, whereas for Cobb-Douglas technology complete foreclosure occurs when firm S's cost share is low ($0 < \alpha < 0.15$), for the CES technology it occurs for intermediate values ($0.26 < s < 0.31$). Intuitively, when the cost share is low, withholding the input would not raise firm 2's cost sufficiently to induce firm 2 to exit the market, and when the share is high the merged firm could raise firm 2's cost sufficiently but prefers to continue to supply firm 2.

Despite these differences, the key finding from the Cobb-Douglas case carries over: the merger can reduce consumer welfare and/or total welfare, unlike in the case without input substitution.[29]

[29] However, a reduction in CW implies a reduction in TW, while with Cobb-Douglas a reduction in TW implies a reduction in CW.

5. Concluding Remarks

We analyzed the effects of a vertical merger between a monopolist input supplier and a downstream firm engaged in duopoly competition with a rival that supplies a differentiated substitute good, when both firms can substitute imperfectly from the monopoly input. Our main model considered Cobb-Douglas technologies (identical or different), and an extension considered a common CES technology. Linear pricing was assumed throughout, hence the merged firm generally has three different pricing incentives compared to the separate merger partners: to decrease the price of the firm's final good in order to reflect the true resource cost of that input (EDM incentive), to increase that same price in order to reflect the opportunity cost from lost input sales to the rival due to sales diverted from its good (Chen incentive), and to raise the input price to the rival (RRC incentive). The EDM and Chen incentives work in opposite directions, and the actual changes in the equilibrium values of all prices will reflect the joint effect of all three incentives.

Somewhat surprisingly, with input substitution—Cobb-Douglas or CES technologies—the merger can reduce total welfare and consumer welfare, whereas in the same environment (including linear demands), prior work showed that a merger is beneficial if the production technology involves fixed proportions. Relatedly, with identical Cobb-Douglas technologies (yielding a symmetric pre-merger equilibrium) the merged firm raises the rival's cost by more, and welfare harm is greater, when the merged firm's input is used in low rather than high intensity. Both findings derive from the same force. Under fixed proportions technologies, the rival's output yields commensurate input profits to the merged firm, reducing its incentive to handicap the rival. With input substitution, the merged firm has a stronger incentive to divert output to its own good by raising the input price and rival's cost and this incentive grows as the rival uses its input less intensively. When firms use different Cobb-Douglas technologies, the merger effects can be complex and non-monotonic in the usage intensity of either firm.

We caution that our analysis involved specific examples and therefore only illustrates possibilities. Future work could explore the robustness of our findings to alternative assumptions about demand and technology. Nevertheless, the analysis does offer two implications for assessing vertical mergers. It cautions against using the ability of a rival to substitute the merged firm's input with other inputs as an indicator of reduced foreclosure concerns. And it reiterates the need to consider the merged firm's profits from sales of inputs to rivals, not only from its own output.

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